SOLUTIONS & ANSWERS FOR KERALA ENGINEERING ENTRANCE EXAMINATION-2018 – PAPER II VERSION – B1

[MATHEMATICS]

1. Ans: $\frac{10}{\sqrt{2}}(1+i)$

Sol: It is 10(cos45° + isin45°) = $\frac{10}{\sqrt{2}}(1+i)$

2. Ans: $\frac{-1}{1+i}$

Sol: $z = \frac{1}{-i-1} = \frac{-1}{1+i}$

3. Ans: 2 – 2i

Sol: The expression = $(-1-i)^3 = \left(-\sqrt{2}\right)^3 \left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

$= -2\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

$= 2(1-i)$

4. Ans: 2

Sol: The number is $\frac{4i}{2} = 2i$; $|2i| = 2$

5. Ans: –1

Sol: $z = \cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}$

$= \cos\left(\pi+\frac{\pi}{3}\right)+i\sin\left(\pi+\frac{\pi}{3}\right)$

$= -\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}$

$= -\frac{1}{2}-i\times\frac{\sqrt{3}}{2} = \omega$ a cube root of unity

$\therefore z^{192} + z^{194} = \omega^{192} + \omega^{194} = 1 + \omega^2 = -\omega$

$\therefore (z^{192} + z^{194})^3 = (-\omega)^3 = -1$

6. Ans: –i – 3

Sol: b + ia = i(a – ib)

$(b + ia)^{11} = -i\,(a - ib)^{11}$

$= -i\,(1 - 3i) = -3 - i$

7. Ans: $3x^2 - 19x + 3 = 0$

Sol: α, β are roots of $x^2 - 5x + 3 = 0$

$\therefore \alpha + \beta = 5, \alpha\beta = 3$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{25-6}{3} = \frac{19}{3}$

$\frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} = 1$

equation required is $x^2 - \frac{19}{3}x + 1 = 0$

$3x^2 - 19x + 3 = 0$

8. Ans: $\left(\frac{-3}{4}, 2\right)$

Sol: Equation is $(y-2)^2 = x + 1$

i.e. $Y^2 = 4aX$ where $X = x + 1$

$Y = y - 2$

$a = \frac{1}{4}$

$\therefore$ focus is $x + 1 = \frac{1}{4}$, $y - 2 = 0$

i.e. $\left(\frac{-3}{4}, 2\right)$

9. Ans: 1

Sol: $x \to 2018 \Rightarrow x > 0$

$\therefore x < 2x < 3x$

$\Rightarrow f(x) < f(2x) < f(3x)$

$\Rightarrow 1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$

$\Rightarrow \lim_{x\to2018} 1 = \lim_{x\to2018}\frac{f(3x)}{f(x)} = 1$

$\therefore$ By squeeze theorem,

$\lim_{x\to2018}\frac{f(2x)}{f(x)} = 1$

10. Ans: 2f(1) – f'(1)

Sol: $\lim_{x\to1}\frac{x^2f(1)-f(x)}{x-1} = \lim_{x\to1}\frac{2x\,f(1)-f'(x)}{1}$

11. Ans: $\frac{\sqrt{3}}{2}$

Sol: Equation is $4(x-1)^2 + (y+2)^2 = 16$

$\frac{(x-1)^2}{4}+\frac{(y+2)^2}{16} = 1$

$e = \sqrt{\frac{16-4}{16}} = \frac{\sqrt{3}}{2}$

12. Ans: $(-4, -1)$

Sol: Equation is $Y^2 = 4aX$
where $Y = y + 1$, $X = -x - 2$
$a = 2$
focus is $-x - 2 = 2$, $y + 1 = 0$
i.e. $(-4, -1)$

13. Ans: $x^2 - y^2 + 3x - 2y - 43 = 0$

Sol: D

14. Ans: 3

Sol: $25p + 5q + r = -3\,(4p + 2q + r)$
$37p + 11q + 4r = 0$
$16p - 4q + r = 0$
$64p - 16q + 4r = 0$
$27q = 27p \Rightarrow \dfrac{q}{p} = 1$
$\therefore$ If α is the root,
$\alpha - 4 = -1 \Rightarrow \alpha = 3$

15. Ans: $\dfrac{1}{4}$

Sol: $f(xy) = f(x)\ f(y)$
$\therefore f(x) = x^n$
$f(2) = 4 \Rightarrow 4 = 2^n \Rightarrow n = 2$
$\therefore f(x) = x^2$
$\therefore f\left(\dfrac{1}{2}\right) = \dfrac{1}{4}$

16. Ans: 2^{58}

Sol: Sum of last 30 coefficients = sum of 1st 30 coefficients
$= \dfrac{1}{2}$ sum of all coefficients
$= \dfrac{1}{2} \cdot 2^{59} = 2^{58}$

17. Ans: $40\sqrt{6}$

Sol: It is $2\left[{}^4C_1\left(\sqrt{3}\right)^3 . \sqrt{2} + {}^4C_3\left(\sqrt{3}\right)\left(\sqrt{2}\right)^3\right]$
$= 2\left(12\sqrt{6} + 8\sqrt{6}\right)$
$= 40\sqrt{6}$

18. Ans: $\dfrac{3}{4}$

Sol: $P(A \cup B \cup C)$
$= \sum P(A) - \sum P(B \cap C) + P(A \cap B \cap C)$
$= \dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{4} - \left(\dfrac{1}{12} + \dfrac{1}{8} + \dfrac{1}{6}\right) + \dfrac{1}{24}$
$= \dfrac{6+4+3}{12} - \dfrac{2+3+4}{24} + \dfrac{1}{24}$
$= \dfrac{13}{12} - \dfrac{4}{12} = \dfrac{3}{4}$

19. Ans: NA

Sol: $\left|\dfrac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right| = 2\sqrt{2}$

20. Ans: 1

Sol: It is $\lim\limits_{x \to \infty} \sqrt{\dfrac{1 - \dfrac{\sin x}{x}}{1 + \dfrac{\cos^2 x}{x}}} = 1$

21. Ans: $\dfrac{\sqrt{3}}{2}$

Sol: $\sin\dfrac{31\pi}{3} = \sin\left(10\pi + \dfrac{\pi}{3}\right) = \sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

22. Ans: $(1001)^2$

Sol: 2001 is the 1001th odd no
$\therefore$ required sum $= 1001^2$

23. Ans: $-\cos 2x$

Sol: $y = \dfrac{\sin^3 x}{\sin\lambda + \cos x} + \dfrac{\cos^3 x}{\sin x + \cos x}$
$= \sin^2 x + \cos^2 x - \sin x \cos x$
$= 1 - \dfrac{1}{2}\sin 2x$
$y' = -\cos 2x$

24. Ans: $\left(\dfrac{24 \pm 5\sqrt{145}}{12}, 1\right)$

Sol: $16(x-2)^2 - 9(y-1)^2 = 145$
Foci are $x - 2 = \pm\dfrac{5\sqrt{145}}{12}$, $y - 1 = 0$
i.e. $\left(\dfrac{24 \pm 5\sqrt{145}}{12}, 1\right)$

25. Ans: 1

Sol: Thus $a^2 - 2a + 1 = 0 \Rightarrow a = 1$

26. Ans: 2.57

Sol: Mean = 6
$M\,D = \dfrac{4+3+3+3+3+2}{7} = \dfrac{18}{7}$
$= 2.57$

27. Ans: $\dfrac{1}{2^{12}}$

Sol: '~p' = 8, 'npq' = 4

$\Rightarrow q = \frac{1}{2},\ p = \frac{1}{2};\ n = 16$

$P(X = 1) = {}^{16}C_1 \left(\frac{1}{12}\right)^{16} = \frac{2^4}{2^{16}} = \frac{1}{2^{12}}$

28. Ans: 90

Sol: The number is ${}^{15}C_2 - 15 = 90$

29. Ans: $\frac{2}{3}$

Sol: P(MS) = 0.4, P(M) = 0.6

$P(S/M) = \frac{P(MS)}{P(M)} = \frac{2}{3}$

30. Ans: $\frac{1}{\sqrt{2}}$

Sol: $(x-1)^2 + 2\left(y + \frac{3}{4}\right)^2 = 1 + \frac{9}{8} - 2$

$$\frac{(x-1)^2}{\frac{1}{8}} + \frac{\left(y+\frac{3}{4}\right)^2}{\frac{1}{16}} = 1$$

$$e^2 = \frac{\frac{1}{8} - \frac{1}{16}}{\frac{1}{8}} = \frac{1}{2}$$

$$e = \frac{1}{\sqrt{2}}$$

31. Ans: 42

Sol: $\sum x_i = 20 \times 10 = 200$

New sum = 200+ (4 + 8 + ...+ ....+ 40)

= 200 + 5(4 + 40)

= 200 + 220

= 420

$\therefore\ \bar{x} = \frac{420}{10} = 42$

32. Ans: $\frac{19}{20}$

Sol:

	S	T	A	I	C	N	
I	3	3	1	2	1		$0/10$
II	3	2	2	1	0		$1/9$

Required probability

$$= \frac{3}{10}.\frac{3}{9} + \frac{3}{10}\times\frac{2}{9} + \frac{1}{10}\times\frac{2}{9} + \frac{2}{10}\times\frac{1}{9}$$

$$= \frac{9+6+2+2}{90} = \frac{19}{20}$$

33. Ans: $a^2 - b^2 + 2ac = 0$

Sol: $\sin\alpha + \cos\alpha = \frac{-b}{a},\ \sin\alpha\cos\alpha = \frac{c}{a}$

$\therefore\ \frac{b^2}{a^2} - 2\frac{c}{a} = \sin^2\alpha + \cos^2\alpha = 1$

$b^2 - 2ac = a^2$

i.e. $a^2 - b^2 + 2ac = 0$

34. Ans: $\frac{15}{4}\sqrt{7}$

Sol: area = $\sqrt{\frac{15}{2}.\frac{7}{2}.\frac{5}{2}.\frac{3}{2}}$

$= \frac{15}{4}\sqrt{7}$

35. Ans: 209

Sol:

B	G
1	3
2	2
3	1
4	0

Required number = ${}^6C_1 \times {}^4C_3 + {}^6C_2 \times {}^4C_2 + {}^6C_3 \times {}^4C_1 + {}^6c_4$

= = 24 + 90 +80 +15 = 209

36. Ans: 720

Sol: There are 10 distinct letters

$\therefore$ required number = ${}^{10}P_3 = 720$

37. Ans: 5

Sol: $5^{97} = 5 \times 25^{48}$

$= 5(26-1)^{48} = 5[(M(52) + 1]$

$\therefore$ remainder is 5

38. Ans: $\frac{1}{3}$

Sol: $b^2 - 4ac \geq 0$

b cannot be 2 or 3

There are only two cases where roots are real

Total number of cases = 3! = 6

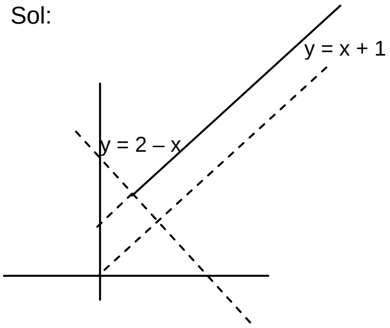
$\therefore$ required probability = $\frac{2}{6} = \frac{1}{3}$

39. Ans: $\frac{3}{4}$

Sol: $\lim_{x\to\infty} \frac{3x^3 + 2x^2 - 7x + 9}{4x^3 + 9x - 2} = \lim_{x\to\infty} \frac{3x^3}{4x^3} = \frac{3}{4}$

40. Ans: $\frac{3}{2}$

Sol:



Minimum where y = 2 – x & y = x + 1 meet
i.e. x + 1 = 2 – x
2x = 1
$x = \frac{1}{2}$
when $x = \frac{1}{2}$, $y = \frac{3}{2}$
$\therefore$ required minimum value = $\frac{3}{2}$

41. Ans: x = 2, y = –3

Sol: Equation is x(y + 3) – 2(y + 3) = 4
i.e. (x – 2) (y + 3) = 4
$\therefore$ Asymptotes are x – 2 = 0, y + 3 = 0
i.e. x = 2, y = –3

42. Ans: $6x^5 + 6^x \log(6)$

Sol: $f'(x) = 6x^5 + 6^x \log 6$

43. Ans: NA

Sol: $\bar{x} = \frac{63}{7} = 9$

$$\frac{\sum (x-\bar{x})^2}{n} = \frac{9+16+0+16+9+1+1}{7}$$

$$= \frac{52}{7}$$

44. Ans: $\frac{m^2}{n^2}$

Sol: $$\lim_{x\to 0} \frac{2\sin^2 \frac{mx}{2}}{2\sin^2 \frac{nx}{2}} = \left[\frac{\left(\frac{m}{2}\right)}{\left(\frac{n}{2}\right)}\right]^2 = \frac{m^2}{n^2}$$

45. Ans: 1

Sol: $$\lim_{x\to 0} \frac{2x}{x\left(\sqrt{1+2x}+1\right)} = \lim_{x\to 0} \frac{2}{\sqrt{1+2x}+1} = 1$$

46. Ans: 12

Sol: h'(x) = f'(g(x)) . g'(x)
h'(3) = f '(7) g'(3)
$= 2 \times 6 = 12$

47. Ans: 4

Sol: It is $\frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ}$

$$= \frac{4\left[\cos\left(30^\circ + 50^\circ\right)\right]}{\sin 40} = \frac{4\cos 50^\circ}{\cos 50^\circ} = 4$$

48. Ans: $\frac{4}{45}e^{-2}$

Sol: $e^{-m}\frac{m^x}{x!}$ is the probability density

$P(X = 1) = e^{-m} m$

$P(X = 2) = e^{-m}\frac{m^2}{2!}$

$P(X = 6) = e^{-m}\frac{m^6}{6!}$

$\therefore \frac{m}{2} = 1 \Rightarrow m = 2$

$P(X = 6) = e^{-2}\frac{2^6}{6!} = \frac{4}{45}e^{-2}$

49. Ans: 1

Sol: The selected numbers can be
(1, 2) or (2, 3) or (3, 4) or ...... or (19, 20)
$\therefore$ Total number of cases = 19
Now,
$a^2 + b^2 + a^2b^2 = (a – b)^2 + 2ab + a^2b^2$
$= 1 + 2ab + a^2b^2$
$= (ab + 1)^2$
$= (2k + 1)^2$, since the product of two consecutive integers is even.
$\therefore \sqrt{a^2+b^2+a^2b^2} = |2k + 1|$, odd integer
$\therefore$ Number of favourable cases = 19
$\therefore$ Probability = 1

50. Ans: $\frac{2}{3}$

Sol: If 'a', 'b' are the semi axes of the ellipse & a > b the circle is or radius a
$\therefore$ required probability = $1 - \frac{\pi ab}{\pi a^2}$

$= 1 - \frac{b}{a}$

$$=1-\sqrt{1-e^2}$$

$$= 1-\sqrt{1-\frac{8}{9}} = \frac{2}{3}$$

51. Ans: 10

Sol: $\begin{vmatrix} 4 & 11 & m \\ 7 & 2 & 6 \\ 1 & 5 & 4 \end{vmatrix} = 0 \Rightarrow 4 \times (-22) - 11$

$22 + m \times 33 = 0$

$\Rightarrow -8 - 22 + 3m = 0$

$\Rightarrow 3m = 30$

$\Rightarrow m = 10$

52. Ans: $4\sqrt{6}$

Sol: $\bar{a}+\bar{b} = 2\bar{i}+4\bar{j}+6\bar{k} = 2(\bar{i}+2\bar{j}+3\bar{k})$

$\bar{b}+\bar{c} = 8\bar{i}+12\bar{j}+16\bar{k} = 4(2\bar{i}+3\bar{j}+4\bar{k})$

$(\bar{a}+\bar{b})\times(\bar{b}+\bar{c}) = -\begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{vmatrix}$

$-8(i - 2j + k)$

$\therefore$ required area $= \frac{1}{2}\times 8\times\sqrt{6}$

$= 4\sqrt{6}$

53. Ans: –29

Sol: $0 = \bar{a}^2+\bar{b}^2+\bar{c}^2+2(\bar{b}.\bar{c}+\bar{c}.\bar{a}+\bar{a}.\bar{b})$

$= 9 + 1 + 16 + 2\sum \bar{b}.\bar{c}$

$\therefore \sum \bar{b}.\bar{c} = -13$

54. Ans: $\frac{\pi}{3}$

Sol: $1 = |\bar{a}|^2 + |\bar{b}|^2 - 2\bar{a}.\bar{b}$

$= 1 + 1 - 2\cos\theta$

$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

55. Ans: –9

Sol: $\lambda - 9 + 2\mu = 0$

$2\lambda + 36 + \mu = 0$

$\lambda + 2\mu = 9$

$2\lambda + \mu = -36$

$3\lambda = -81 \Rightarrow \lambda = -27$

$\mu = 18$

$\lambda + \mu = -9$

56. Ans: –1024 , 1

Sol: $\left(x^{\frac{1}{5}}+4\right)\left(x^{\frac{1}{5}}-1\right) = 0$

$x^{\frac{1}{5}} = 1 \Rightarrow x = 1$

$x^{\frac{1}{5}} = -4 \Rightarrow x = (-4)^5 = -1024$

$\therefore x = -1024$ or 1

57. Ans: –1

Sol: $b^2 + ab + 1 = 0$, $b^2 - b - a = 0$

$ab + a + b + 1 = 0 \Rightarrow (a + 1)(b + 1) = 0$

$\Rightarrow a = -1$ or $b = -1$

$a = -1$ gives imaginary roots for $x^2 - x - a = 0$

58. Ans: 1

Sol: $1 - 2\sin\theta\cos\theta = 1$

$\sin\theta\cos\theta = 0$

$\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)$

$(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)$

$= (\sin\theta - \cos\theta)(1 + 0)$

$= 1 . 1 = 1$

59. Ans: $\frac{2}{9}$

Sol: $\left.\begin{matrix} 1 & 6 \\ 2 & 5 \\ 3 & 4 \end{matrix}\right\}\times 2 \quad 6 \quad 5\}\times 2$

Required probability $= \frac{8}{36} = \frac{2}{9}$

60. Ans: $\frac{2^9}{10\,!}$

Sol: $\frac{1}{9i}\left(2+24+\frac{126}{5}\right)$

$= \frac{1}{9i}\times\frac{256}{5}$

$= \frac{2^8}{9!\times 5} = \frac{2^9}{10\,!}$

61. Ans: 3 and 2

Sol: order – 3

degree –2

62. Ans:4

Sol: $2\int_0^2 |x|\,dx = 2\left(\frac{x^2}{2}\right)_0^2$

$= 2(2 - 0) = 4$

63. Ans: $\frac{\pi}{4}$

Sol: $\int_{-1}^{0}\frac{dx}{(x+1)^2+1} = \left(\tan^{-1}(x+1)\right)_{-1}^{0}$

$= \tan^{-1}1 - \tan^{-1}0$

$= \frac{\pi}{4}$

64. Ans: 5

Sol: $\int_{-1}^{4} f(x)dx = 4$

$\Rightarrow \int_{-1}^{2} f(x)dx + \int_{2}^{4} f(x)dx = 4$

$\Rightarrow \int_{-1}^{2} f(x)dx + -1 = 4$

$\left(\because \int_{2}^{4}(3-f(x))dx = 7\right.$

$\therefore \int_{2}^{4} f(x)dx = 6-7 = -1$

$\therefore \int_{-1}^{2} f(x)dx = 4+1 = 5$

65. Ans:

Sol: $\int \frac{(x+1-1)}{(x+1)^2}e^x\ dx$

$= \int\left(\frac{1}{x+1} - \frac{1}{(x+1)^2}\right)e^x\ dx$

$= \frac{e^x}{1+x} + C$ $\qquad = \int e^x\left(f(x)+f'(x)\right)dx$

$= e^x f(x) + C$

66. Ans: 1

Sol: $2^{2000} = (16+1)^{500}$

$\Rightarrow$ Remainder = 1

67. Ans: 1512

Sol: Coefficient of $x^5 = {}^8C_3\ 3^3$

$= 1512$

68. Ans: 10

Sol: $\left(5+\frac{3}{2}\right)\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$\frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

Maximum value = $C + \sqrt{a^7 + b^2}$

$= 3 + \sqrt{\frac{169}{4} + \frac{27}{4}}$

$= 3 + \frac{14}{2} = 3 + 7 = 10$

69. Ans: $\frac{1}{2}|z|^2$

Sol: Area = $\frac{1}{2}|z|^2$

70. Ans: 0

Sol: f(x) = cosx f'(x) = –sinx

$\therefore$ f '(0) = 0

71. Ans: $\left(\frac{19}{3}, \frac{8}{3}\right)$

Sol: By section formula $\left(\frac{56+20}{12}\right), \left(\frac{42-10}{12}\right)$

$\Rightarrow \left(\frac{19}{3}, \frac{8}{3}\right)$

72. Ans: 0

Sol: A = $\frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$

$= \frac{a+b+c}{2}\begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{vmatrix} = 0$

73. Ans: bx – ay = 0

Sol: Midpoint (a, b) satisfy only bx – ay = 0

74. Ans: $\frac{x}{a} + \frac{y}{b} = 2$

Sol: Equation line of line bx + ay = ab
any line parallel to bx +ay = 2ab
passing through (a, b), k = 2ab

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 2$

75. Ans: (8, 8)

Sol: Area = $\frac{1}{2}\begin{vmatrix} 2a & a & 1 \\ a & 2a & 1 \\ a & a & 1 \end{vmatrix} = 18$

$= \frac{1}{2}a^2\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 18$

$\Rightarrow \frac{1}{2}a^2(1) = 18$

$\Rightarrow a^2 = 36$

$\therefore a = \pm 6$

$\therefore$ centroid $\left(\frac{4a}{3}, \frac{4a}{3}\right)$ = (8, 8)

76. Ans: $\left(\frac{-1}{4}, \frac{11}{4}\right)$

Sol: $\begin{vmatrix} 2 & 1 & 1 \\ 3 & -2 & 1 \\ a & a+3 & 1 \end{vmatrix} = \pm 5$

$2(-2 - a - 3) - 1(3 - a) + 1(3a + 9 + 2a) = \pm 5$

$-10 - 2a - 3 + a + 5a + 9 = \pm 5$

$4a - 4 = \pm 5$

$4a = \pm 5 + 4 = 9$ or -2

$\Rightarrow a = 14$ or $\frac{-1}{4}$

vertex is (a, a + 3)

$= \left(\frac{-1}{4}, \frac{-1}{4} + 3\right) = \left(\frac{-1}{4}, \frac{11}{4}\right)$

77. Ans: 1

Sol: For pair of straight line

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow 1 + 0 - f^2 - g^2 = 0$

$\Rightarrow f^2 + g^2 = 1$

78. Ans: $\frac{9}{25}$

Sol: $\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} = \frac{2\sqrt{\frac{25}{4} - 4}}{1+4}$

$= \frac{2\left(\frac{3}{2}\right)}{5} = \frac{3}{5}$

$\tan^2\theta = \frac{9}{25}$

79. Ans: x = 3, y = 1, z = 2

Sol: $3 = 2x + y - 2z$

$2 = -x + 3y - 2 + z$

$-5 = x + -2y - 3z$

back substitution

80. Ans: $\frac{\sqrt{3} - 1}{2\sqrt{2}}$

Sol: sin(45 – 30°)

$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$

81. Ans: $i + 2j + 2\hat{k}$

Sol: $\bar{b} = 3\hat{i} + 6\hat{j} + 6\hat{k}$

$\bar{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$\bar{a}$ & $\bar{b}$ collinear

$\bar{a}.\bar{b} = 27$

82. Ans: $\frac{65}{13}\sqrt{133}$

Sol: $\left|\bar{a} \times \bar{b}\right|^2 = |a|^2\,|b|^2 - (a.\,b)^2$

$|a \times b|^2 = 169.\,25 - 900$

$|a \times b| = \sqrt{3325} = \frac{65}{13}\sqrt{133}$

$= 5\sqrt{133}$

83. Ans: 41

Sol: ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$

$\frac{\frac{56!}{(56 - r - 6)!}}{\frac{54!}{(54 - r - 3)!}} = 30800$

$51 - r = 10$

$41 = r$

84. Ans: $\sqrt{5}$

Sol: $2x - y + 4 = 0$

$2x - y - 1 = 0$

$d = \frac{|-1 - 4|}{\sqrt{4+1}} = \frac{5}{\sqrt{5}} = \sqrt{5}$

85. Ans: 2^7

Sol: ${}^7C_0 + 2\,.\,{}^7C_1 + 2^7C_3 + {}^7C_7 + 2\,{}^7C_2$

$1 + 14 + 70 + 1 + 42$

$= 128 = 2^7$

86. Ans: –19

Sol: $(1 - 3x + 7x^2)(1 - x)^{16}$

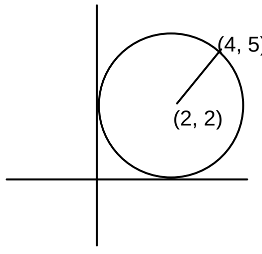
coefficient of x

$1(-{}^{16}C_1x) + -3x\,({}^{16}C_0\,x^0)$

$= -16x - 3x = -19x$

87. Ans: $x^2 + y^2 - 4x - 4y - 5 = 0$

Sol:



$r = \sqrt{4+9}$

$(x-h)^2 + (y-k)^2 = r^2$
$(x-2)^2 + (y-2)^2 = 13$
$x^2 - 4x + 4 + y^2 - 4y + 4 = 13$
$x^2 + y^2 - 4x - 4y - 5 = 0$

88. Ans: (3, 2, 0)

Sol: Let P(x, y, 0) be a point on xy plane

P(x, y, 0) $\quad A \Rightarrow (2, 0, 3)$
$B = (0, 3, 2)$
$C \Rightarrow (0, 0, 1)$

$PA^2 = (2-x)^2 + y^2 + 9$
$PB^2 = x^2 + (3-y)^2 + 4 \quad PA^2 = PB^2 = PC^2$
$PC^2 = x^2 + y^2 + 1$
$(2-x)^2 + 9 = x^2 + 1 \; (PA^2 = PC^2)$
$4 + x^2 - 4x + 9 = x^2 + 1 - 13 \; (PA^2 = PB^2)$
$-4x = -12 \quad -6y = -12$
$y = 2$
$x = 3$

89. Ans: 3

Sol: f(1) = 2 and f(0) = 1
f(x + y) = f(x) f(y) gives $f(x) = 2^x$

$\therefore \sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$

$\Rightarrow 2^{a+1} + 2^{a+2} + 2^{a+3} + \ldots + 2^{a+n} = 16(2^n - 1)$

$\Rightarrow 2^a (2 + 2^2 + \ldots + 2^n) = 16(2^n - 1)$

$\Rightarrow 2^a \; 2\left(\frac{2^n - 1}{2 - 1}\right) = 16(2^n - 1)$

$\Rightarrow 2^{a+1} (2^n - 1) = 16(2^n - 1)$

$\therefore 2^{a+1} = 16$

$\Rightarrow a + 1 = 4 \Rightarrow a = 3$

90. Ans: 9

Sol: ${}^nC_{r-1} = 36$ —— (1)
${}^nC_r = 84$ —— (2)
${}^nC_{r+1} = 126$ —— (3)

$\frac{(2)}{(1)} \Rightarrow 3n - 10r = -3$ —— (4)

$\frac{(3)}{(2)} \Rightarrow 2n - 5r = 3$ —— (5)

Solving (4) and (5)
n = 9

91. Ans: 2

Sol: $f(x) = f(x^2) \; \forall x \in (-1, 1)$
$\Rightarrow$ f(x) is a constant

$\therefore f(x) = \frac{1}{2} \; \forall x$

$\therefore 4f\left(\frac{1}{4}\right) = 4 \times \frac{1}{2} = 2$

92. Ans: 0

Sol: $\lim_{x \to \infty} \frac{(x^2+1) - (x^2-1)}{\sqrt{x^2+1} + \sqrt{x^2-1}}$

$= \lim_{x \to \infty} \frac{2}{x\sqrt{1+\frac{1}{x^2}} + \sqrt{1-\frac{1}{x^2}}} = 0$

93. Ans: 5

Sol: $f'(1) = \lim_{x \to 0} \frac{f(1+h) - f(1)}{h}$

$= \lim_{x \to 0} \left[5 - \frac{f(1)}{h}\right]$

$\therefore$ limit exist $\Rightarrow f(1) = 0$
$\Rightarrow f'(1) = 5$

94. Ans: 2

Sol: $f'(x) = 0 \Rightarrow 6x^2 - 30x + 36 = 0$
$= 6(x^2 - 5x + 6) = 0$
$\Rightarrow x = 3, 2$
$= f'(x) < 0$ at x = 2
$\therefore$ Maximum at x = 2

95. Ans:

Sol: $\int f(x) \cos x dx = \frac{[f(x)]^2}{2} + C$

$\Rightarrow f(x) = \sin x + C$

$f\left(\frac{\pi}{2}\right) = 1 + C$

96. Ans: $\pi(\sqrt{2} - 1), \; \frac{\pi}{\sqrt{2} + 1}$

Sol: $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1 + \sin x} dx$

Also $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi - x}{1 + \sin(\pi - x)} dx = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi - x}{1 + \sin x}$

$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi}{1 + \sin x} dx = \pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1 - \sin x}{\cos^2 x} dx$

$= \pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sec^2 x - \sec x \tan x) dx$

$= \pi [\tan x - \sec x]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$

$= \pi\left[\left(\tan\frac{3\pi}{4}-\tan\frac{\pi}{4}\right)-\left(\sec\frac{3\pi}{4}-\sec\frac{\pi}{4}\right)\right]$

$= \pi\left[(-1-1)-\left(-\sqrt{2}-\sqrt{2}\right)\right]$

$= \pi\left[-2+2\sqrt{2}\right] = 2\pi\left(\sqrt{2}-1\right)$

$\therefore I = \pi\left(\sqrt{2}-1\right)$

97. Ans: $\frac{\pi}{4}$

Sol: $I = \int_0^{\frac{\pi}{2}} \frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}dx$ ——— (1)

$I = \int_0^{\frac{\pi}{2}} \frac{2^{\cos x}}{2^{\sin x}+2^{\cos x}}dx$ ——— (2)

(1) + (2)

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1dx$

$2I = \frac{\pi}{2}$

$I = \frac{\pi}{4}$

98. Ans: 0

Sol: $\lim_{x\to 0}\left(\frac{\int_a^{x^2}\sin\sqrt{t}\ dt}{x^2}\right)\left(=\frac{0}{0}\right)$

$\lim_{x\to 0}\left(\frac{\sin x.\ \ 2x}{2x}\right) = 0$

99. Ans: $\frac{\pi}{4}$

Sol: Area = $\int_{\frac{\pi}{2}}^{\pi}\sin^2 x dx = \int_{\frac{\pi}{2}}^{\pi}\left(\frac{1-\cos 2x}{2}\right)dx$

$= \frac{\pi}{4}$ sq. units

100. Ans: y" – 2y' + 2y = 0

Sol: The solution is y" – 2y' + 2y = 0

101. Ans: NA

Sol: $i-\sqrt{3} = i\left[1+i\sqrt{3}\right] = 2i\left[\frac{1}{2}+i\frac{\sqrt{3}}{2}\right]$

$= -2i\ \omega^2$

$\left(i-\sqrt{3}\right)^{13} = (-2i)^{13}\ \omega^{26}$

$= (-2)^{13}\ i^{13}\ \omega^{26}$

$= (-2)^{13}\ i\ \omega^2$

$= (-2)^{13}\ i\left[\frac{-1}{2}-\frac{i\sqrt{3}}{2}\right]$

Real part = $(-2)^{13}\times\frac{\sqrt{3}}{2}$

102. Ans: $\frac{-1}{2}$

Sol: $\lim_{x\to 0}\frac{1+x-e^x}{x^2}\left(\frac{0}{0}\right)$

Applying L- Hospitals rule

$\lim_{x\to 0}\frac{1-e^x}{2x}\left(\frac{0}{0}\right)$

Applying L- Hospital's rule again

$\lim_{x\to 0}\frac{-e^x}{2} = \frac{-1}{2}$

103. Ans: $\frac{\sin x-\cos x}{\sin 2x}+C$

Sol: $\int\frac{(\sin x+\cos x)(2-\sin 2x)}{\sin^2 2x}dx$

$= \int\frac{(\sin x+\cos x)[1+(1-\sin 2x)]}{[1-(1-\sin 2x)]^2}dx$

$= \int\frac{(\sin x+\cos x)\left[1+\left(\begin{matrix}\sin^2 x+\cos^2 x-\\ -2\sin x\cos x\end{matrix}\right)\right]dx}{\left[1-\left(\sin^2 x+\cos^2 x-2\sin x\cos x\right)\right]^2}dx$

$= \int\frac{(\sin x+\cos x)\left[1+(\sin x-\cos x)^2\right]dx}{\left[1-(\sin x-\cos x)^2\right]^2}$

$= \int\frac{1+t^2}{\left(1-t^2\right)^2}dt$, where t = sinx – cosx

$\int\frac{\left(1-t^2\right)+2t}{\left(1-t^2\right)^2}dt$

$= \int\left[\frac{1}{(1+t)^2}+\frac{2t}{\left(1-t^2\right)^2}\right]dt = \frac{-1}{1+t}+\frac{1}{1-t^2}+C$

$= \frac{t}{1-t^2}+C = \frac{\sin x-\cos x}{1-(\sin x-\cos x)}+C$

$= \frac{\sin x-\cos x}{\sin 2x}+C$

104. Ans: $\bar{r}.\left(2\bar{i}+\bar{j}+2\bar{k}\right) = 15$

Sol: $\bar{n} = \frac{2\bar{i}+\bar{j}+2\bar{k}}{\sqrt{2^2+1^2+2^2}} = \frac{1}{3}\left(2\bar{i}+\bar{j}+2\bar{k}\right)$

Equation is $\bar{r}.\ \bar{n} = d$

$\bar{r}.\ (2\bar{i} + \bar{j} + 2\bar{k}) = 15$

105. Ans: $\tan\frac{A-B}{2}$

Sol: $\frac{\sin A - \sin B}{\cos A + \cos B} = \frac{\frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{2}}{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}$

$= \tan\frac{A-B}{2}$

106. Ans: –16x

Sol: x = Acos4t + Bsin4t

$\frac{dx}{dt} = -4A\sin 4t + 4B\cos 4t$

$\frac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t$

= –16x

107. Ans: $\frac{2^n}{n+1}$

Sol: A.M = $\frac{C_0 + {}^nC_1 + \ldots\ldots + C_n}{n+1} = \frac{2^n}{n+1}$

108. Ans: $\frac{133}{4}$

Sol: Variance of first 20 natural number is

$= \frac{x^2 - 1}{12}$

$= \frac{20^2 - 1}{12} = \frac{399}{12}$

$= \frac{133}{4}$

109. Ans: 90

Sol: number of element in S = 10 × 9 = 90

110. Ans: $\frac{1}{12}$

Sol: S = {H_1, H_2, H_3, H_4, H_5, H_6, T_1, T_2, T_3, T_4, T_5, T_6}

P(coin shows head and die show 3)

$= \frac{1}{12}$

111. Ans: 4

Sol: $|A| = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} = 0 - 1\ (1 - 9) + 2(1 - 6)$

= 8 – 10 = –2

$A_{11} = 2 - 3 = -1$

$A_{22} = 0 - 6 = -6$

$A_{33} = 0 - 1 = -1$

∴ diagonal element of A^{-1} are $\frac{1}{2}, \frac{6}{2}, \frac{1}{2}$

sum = $\frac{1}{2} + \frac{6}{2} + \frac{1}{2} = 4$

112. Ans: 2, 7

Sol: $f(x) = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

$R_1 \to R_1 + R_2 + R_3$

$\begin{vmatrix} x+9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

$x + 9\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$x + 9\begin{vmatrix} 1 & 0 & 0 \\ 2 & x-2 & 0 \\ 7 & 1 & x-7 \end{vmatrix} = 0$

⇒ (x + 9) (x – 2) (x – 7) = 0

⇒ x = –9, 2, 7

113. Ans: –2; –14

Sol: $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

$\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 7+2x \\ 12+x \\ 21+2x \end{bmatrix}$

= [7x+ 2x + 12x + x^2 + 21 + 2x]

= [x^2 + 16x + 28]

∴ x^2 + 16x + 28 = 0

x = –2; –14

114. Ans: $\frac{1}{2}$

Sol: $AA^{-1} = I$

$\Rightarrow \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow x = \frac{1}{2}$

115. Ans: –11

Sol: $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = x(6x-6x)-2\left(6x^2-6x\right)$

$+ x(x^3 - x^2)$

$= 0 - 12x^2 + 12x + x^4 - x^3$

$= x^4 - x^3 - 12x^2 + 12x$

$= ax^4 + bx^3 + cx^2 + dx + e$

$\therefore 5a + 4b + 3c + 2d + e$

$= 5 - 4 - 36 + 24 + 0$

$= -40 + 29 = -11$

116. Ans: 0

Sol: $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

$= \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix} C_3 \to C_3 + C_2$

$= (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} = 0$

117. Ans: 0

Sol: $f(x) = (x-1)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x & x-2 & x \end{vmatrix}$

$= (x-1)\begin{vmatrix} 1 & 0 & 0 \\ 2x & -(x+1) & -x \\ 3x & -2(x+1) & -2x \end{vmatrix} \begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{matrix}$

$= (x-1)(x+1)x\begin{vmatrix} 1 & 0 & 0 \\ 2x & -1 & -1 \\ 3x & -2 & -2 \end{vmatrix} = 0$

$\therefore f(50) = 0$

118. Ans: $\frac{-1}{2}$

Sol: $\Delta(x)$

$= \begin{vmatrix} 0 & \cos x & 1-\cos x \\ 0 & \cos x & 1+\sin x^{-1}\cos x \\ -1 & \sin x & 1 \end{vmatrix} C_1 \to C_1 - C_2 - C_3$

$= (-1)\,[\cos x\,(1+\sin x - \cos x) - \cos x(1-\cos x)]$

$= (-1)\,[\cos x + \sin x\cos x - \cos^2 x - \cos x + \cos^2 x]$

$= -\sin x\cos x$

$\int_0^{\frac{\pi}{2}} \Delta(x)dx = \int_0^{\frac{\pi}{2}} -\sin x\cos x \; dx =$

$\frac{-1}{2}\int_0^{\frac{\pi}{2}} \sin 2x dx$

$= \frac{1}{4}(\cos 2x)_0^{\frac{\pi}{2}}$

$= \frac{1}{4}(-1-1] = \frac{-1}{2}$

119. Ans: $5x + 6y + 2z - 23 = 0$

Sol: Equation is $\begin{vmatrix} x-1 & y-2 & z-3 \\ -2 & 2 & -1 \\ 4 & -3 & -1 \end{vmatrix} = 0$

$(x-1)(-2-3) - (y-2)(2+4) + (z-3)(6-8) = 0$

$-5x + 5 - 6y + 12 - 2z + 6 = 0$

$-5x - 6y - 2z + 23 = 0$

$5x + 6y + 2z - 23 = 0$

120. Ans: 50(507)

Sol: $t_n = 5n + 1$

$t_1 = 6$

$t_{100} = 501$

$S_{100} = \frac{100}{2}[6+501] = 50(507)$